Name of College - S.S. College, J-Bad
Deptt - Mathematics
TOPIC - Resolution of Trigonometrical (Problem)
Functions into Factors
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Period - 7.30 A.M to 9.00 A.M
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B.Sc I (HONS)

Problem । Show that-

$$\cos x - \cot y \sin x =$$
$$\left(1-\frac{x}{y}\right)\left(1-\frac{x}{\pi-y}\right)\left(1-\frac{x}{\pi+y}\right)\left(1+\frac{x}{2\pi-y}\right)$$
$$\left(1-\frac{x}{2\pi+y}\right)\cdots$$

Solution :→

$$L.H.S = \cos x - \cot y \sin x$$
$$= \cos x - \frac{\cos y}{\sin y} \cdot \sin x$$
$$= \frac{\sin y \cos x - \cos y \sin x}{\sin y}$$
$$= \frac{\sin(y-x)}{\sin y}$$

Now $\sin(y-x) = (y-x)\left[1-\frac{(y-x)^2}{\pi^2}\right]\left[1-\frac{(y-x)^2}{2^2\pi^2}\right]\cdots$

$\left[\text{Since } \sin\theta = \theta\left(1-\frac{\theta^2}{\pi^2}\right)\left(1-\frac{\theta^2}{2^2\pi^2}\right)\left(1-\frac{\theta^2}{3^2\pi^2}\right)\cdots\infty\right.$

$$= (y-x)\left\{\frac{\pi^2-(y-x)^2}{\pi^2}\right\}\left\{\frac{2^2\pi^2-(y-x)^2}{2^2\pi^2}\right\}\cdots$$

$$= (y-x)\frac{(\pi-y+x)(\pi+y-x)}{\pi^2}\frac{(2\pi-y+x)(2\pi+y-x)}{2^2\pi^2}\cdots$$

Also $\sin y = y\left[1-\frac{y^2}{\pi^2}\right]\left[1-\frac{y^2}{2^2\pi^2}\right]\cdots$

$$= y\frac{(\pi-y)(\pi+y)}{\pi^2}\frac{(2\pi-y)(2\pi+y)}{2^2\pi^2}\cdots$$

therefore, $\frac{\sin(y-x)}{\sin y} = \frac{y-x}{y}\frac{(\pi-y+x)(\pi+y-x)}{(\pi-y)(\pi+y)}\cdots$

$$\Rightarrow \frac{\sin(y-x)}{\sin y} = \left[\frac{y-x}{y}\right]\left[\frac{\{(\pi-y)+x\}\{\pi+y-x\}}{(\pi-y)(\pi+y)}\right] \cdots\cdots$$

$$= \left(1-\frac{x}{y}\right)\left[1+\frac{x}{\pi-y}\right]\left[1-\frac{x}{\pi+y}\right] \cdots\cdots$$

Show that

$$\sin x + \cos x = \left(1+\frac{4x}{\pi}\right)\left(1-\frac{4x}{3\pi}\right)\left(1+\frac{4x}{5\pi}\right)\left(1-\frac{4x}{7\pi}\right)\cdots$$

L.H.S $= \sin x + \cos x$

using $a\cos\theta \pm b\sin\theta$

$$= \left[\frac{a}{\sqrt{a^2+b^2}}\cos\theta \pm \frac{b}{\sqrt{a^2+b^2}}\sin\theta\right]$$

$$= \left[\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right]\sqrt{2}$$

$$= \frac{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x}{\frac{1}{\sqrt{2}}}$$

$$= \frac{\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}}{\sin\frac{\pi}{4}}$$

$$= \frac{\sin\left(x+\frac{\pi}{4}\right)}{\sin\frac{\pi}{4}} \text{ ———— } (1)$$

Now $\sin\left(x+\frac{\pi}{4}\right) = \left(x+\frac{\pi}{4}\right)\left[1-\frac{(x+\pi/4)^2}{\pi^2}\right]\left[1-\frac{(x+\pi/4)^2}{2^2\pi^2}\right]\left(1-\frac{\left(x+\frac{\pi}{4}\right)^2}{3^2\pi^2}\right)\cdots$

$\Rightarrow$

$$\sin\left(x+\frac{\pi}{4}\right) = \left(x+\frac{\pi}{4}\right)\left[\frac{\pi^2-\left(x+\frac{\pi}{4}\right)^2}{\pi^2}\right]\left[\frac{2^2\pi^2-\left(x+\frac{\pi}{4}\right)^2}{2^2\pi^2}\right]\left[\frac{3^2\pi^2-\left(x+\frac{\pi}{4}\right)^2}{3^2\pi^2}\right] \cdots$$

$$= \left(x+\frac{\pi}{4}\right)\left[\frac{\pi^2-\left(x+\frac{\pi}{4}\right)^2}{\pi^2}\right]\left[\frac{2\pi^2-\left(x+\frac{\pi}{4}\right)^2}{2^2\pi^2}\right]\left[\frac{3^2\pi^2-\left(x+\frac{\pi}{4}\right)^2}{3^2\pi^2}\right] \cdots \quad (2)$$

$$\cancel{\sin\frac{\pi}{4} = \frac{\pi}{4}\left[1-\frac{\frac{\pi^2}{4^2}}{\pi^2}\right]\left[1-\frac{\frac{\pi^2}{4^2}}{2^2\pi^2}\right]\left[1-\frac{\frac{\pi^2}{4^2}}{3^2\pi^2}\right] \cdots}$$

$$\sin\frac{\pi}{4} = \frac{\pi}{4}\left(1-\frac{\frac{\pi^2}{4^2}}{\pi^2}\right)\left(1-\frac{\frac{\pi^2}{4^2}}{2^2\pi^2}\right)\left(1-\frac{\frac{\pi^2}{4^2}}{3^2\pi^2}\right) \cdots$$

$$= \frac{\pi}{4}\left[\frac{\pi^2-\frac{\pi^2}{4^2}}{\pi^2}\right]\left[\frac{2^2\pi^2-\frac{\pi^2}{4^2}}{2^2\pi^2}\right]\left[\frac{3^2\pi^2-\frac{\pi^2}{4^2}}{3^2\pi^2}\right] \cdots \quad (3)$$

(2) $\div$ (3)

$$\frac{\sin\left(x+\frac{\pi}{4}\right)}{\sin\frac{\pi}{4}} = \left(\frac{x+\frac{\pi}{4}}{\frac{\pi}{4}}\right)\frac{\pi^2-\left(x+\frac{\pi}{4}\right)^2}{\pi^2-\frac{\pi^2}{4^2}}\;\frac{2^2\pi^2-\left(x+\frac{\pi}{4}\right)^2}{2^2\pi^2-\frac{\pi^2}{4^2}} \cdots$$

$$= \left(1+\frac{4x}{\pi}\right)\left(\frac{\left(\pi+x+\frac{\pi}{4}\right)\left(\pi-x-\frac{\pi}{4}\right)}{\left(\pi+\frac{\pi}{4}\right)\left(\pi-\frac{\pi}{4}\right)}\right)\frac{\left(2\pi+x+\frac{\pi}{4}\right)\left(2\pi-x-\right.}{\left(2\pi+\frac{\pi}{4}\right)\left(2\pi-\frac{\pi}{4}\right)}$$

$$\Rightarrow \frac{\sin\left(x+\frac{\pi}{4}\right)}{\sin\frac{\pi}{4}} = \frac{\left(1+\frac{4x}{\pi}\right)\left(x+\frac{5\pi}{4}\right)\left(\frac{3\pi}{4}-x\right)\left(\frac{9\pi}{4}+x\right)\left(\frac{7\pi}{4}-x\right)}{\frac{5\pi}{4}\cdot\frac{3\pi}{4}\cdot\frac{9\pi}{4}\cdot\frac{7\pi}{4}}$$

$$= \left(1+\frac{4x}{\pi}\right)\left(1-\frac{4x}{3\pi}\right)\left(1+\frac{4x}{5\pi}\right)\left(1-\frac{4x}{7\pi}\right)\left(1+\frac{4x}{9\pi}\right)$$

Hence the Result